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Neutrino mass matrices leaving no trace

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Abstract

We point attention to the fact that in $SO(10)$ models with non-canonical (type II) see-saw mechanism and exact b–τ unification the trace of the neutrino mass matrix is very small, in fact practically zero. This has the advantage of being a basis independent feature. Taking a vanishing trace as input, immediate phenomenological consequences for the values of the neutrino masses, the CP phases or the predictions for neutrinoless double beta decay arise. We analyze the impact of the zero trace condition for the normal and inverted mass ordering and in case of CP conservation and violation. Simple candidate mass matrices with (close to) vanishing trace and non-zero U_{e3} are proposed. We also compare the results with the other most simple basis independent property, namely a vanishing determinant.

1. Introduction

The neutrino mass matrix m_ν contains more parameters than can be measured in realistic experiments. This concerns in particular the lightest of the three mass eigenstates and one if not both of the Majorana phases [1]. In addition, if the mixing matrix element $|U_{e3}|$ is too small, also the Dirac phase will be unobservable. Thus, the presence of certain conditions or simplifications of the neutrino mass matrix is more than welcome. What comes first to one's mind is of course the presence of zeros in the mass matrix [2]. However, zeros in a certain basis must not be zeros in another one, so that basis independent conditions are advantageous to consider. Any matrix possesses two basis independent quantities, namely its trace and its determinant. The most simple situation is present if these quantities are zero. The condition $\det m_\nu = 0$ [3] leads to one neutrino with vanishing mass and courtesy of this fact one gets also rid of one of the notorious Majorana phases. A vanishing determinant can be motivated on various grounds [4,5]. The second, most simple basis independent requirement is a vanishing trace, i.e., $\mathrm{Tr}\, m_\nu = 0$. Its consequences have first been investigated in [6] applying a three neutrino framework that simultaneously explains the anomalies of solar and atmospheric neutrino oscillation experiments as well as the LSND experiment. In [7], the CP conserving traceless m_ν has been investigated for the more realistic case of explaining only the atmospheric and solar neutrino deficits. Motivation of traceless mass matrices can be provided

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by models in which m_ν is constructed through a commutator of two matrices, as it happens in models of radiative mass generation [8]. More interestingly, and stressed here, a (close to) traceless m_ν can be the consequence of exact b–τ unification at high scale within type II see-saw models [9], which in this framework is also the reason for maximal atmospheric neutrino mixing [10,11]. The type II see-saw mechanism was the original motivation of the traceless m_ν condition as investigated in [6].

In this Letter we shall investigate the outcomes of the requirement $\mathrm{Tr}\, m_\nu = 0$ for the values of the neutrino masses and in case of CP violation also of the CP phases. We investigate the predictions for observables such as the effective mass measured in neutrinoless double beta decay and compare them with the ones stemming from the zero determinant case. Simple forms of m_ν that accomplish the traceless condition and allow for simple correlations between the mixing parameters, mass squared differences and the effective mass as measurable in neutrinoless double beta decay are presented.

2. Framework

2.1. Data

The light neutrino Majorana mass matrix m_ν is observable in terms of

$$m_\nu = U m_\nu^{\mathrm{diag}} U^T. \tag{1}$$

Here m_ν^{diag} is a diagonal matrix containing the light neutrino mass eigenstates m_i. For the normal mass ordering (NH) one has $|m_3| > |m_2| > |m_1|$, whereas the inverted mass ordering (IH) implies $|m_2| > |m_1| > |m_3|$. Mixing is described by U, the unitary Pontecorvo–Maki–Nagakawa–Sakata [12] lepton mixing matrix, which can be parametrized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}\big(1, e^{i\alpha}, e^{i(\beta+\delta)}\big), \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The phases are usually distinguished as the "Dirac phase" δ and the "Majorana phases" [13] α and β. The former can be measured in oscillation experiments, whereas the latter show up only in lepton number violating processes. Their influence on the values of the mass matrix elements is known [14,15], however, only the ee element of m_ν can realistically be expected to be measured [14,16].

In case of CP conservation, different relative signs of the masses m_i are possible, corresponding to the intrinsic CP parities of the neutrinos [17,18]. Four situations are possible, with $m_i = \eta_i |m_i|$ one can write these cases as $(+++)$, $(+--)$, $(-+-)$ and $(--+)$, where the $(\pm\pm\pm)$ correspond to the relative signs of the mass states. Special values of the phases correspond to these sign signatures [18]:

$$\begin{aligned} &(+++) \quad \eta_1 = \eta_2 = \eta_3 = 1 \leftrightarrow \alpha = \beta = \pi, \\ &(+--) \quad \eta_1 = -\eta_2 = -\eta_3 = 1 \leftrightarrow \alpha = \beta = \tfrac{\pi}{2}, \\ &(-+-) \quad \eta_1 = -\eta_2 = \eta_3 = -1 \leftrightarrow \alpha = \tfrac{\beta}{2} = \tfrac{\pi}{2}, \\ &(--+) \quad \eta_1 = \eta_2 = -\eta_3 = -1 \leftrightarrow \alpha = 2\beta = \pi. \end{aligned} \tag{3}$$

Observation implies the following values of the oscillation parameters [19]:

$$\begin{aligned} &\tan^2\theta_{12} = 0.29, \ldots, 0.82, \qquad \sin^2\theta_{13} < 0.05, \qquad \tan^2\theta_{23} = 0.45, \ldots, 2.3, \\ &\Delta m^2_\odot \simeq \big(5.4(14), \ldots, 10(19)\big) \times 10^{-5}\ \mathrm{eV}^2, \qquad \Delta m^2_{\mathrm{A}} \simeq (1.5, \ldots, 3.9) \times 10^{-3}\ \mathrm{eV}^2, \end{aligned} \tag{4}$$

where the 90% C.L. ranges for the respective quantities are given. For $\Delta m^2_\odot$ two upper and lower limits are given, corresponding to the so-called LMA-I and LMA-II solutions [20]. The best-fit points are located in the LMA-I parameter space and are [19] $\tan^2\theta_{12} = 0.45$, $\Delta m^2_\odot = 7.1 \times 10^{-5}$ eV^2. For the atmospheric sector one finds the best-fit points $\tan^2\theta_{23} = 1$ and $\Delta m^2_{\rm A} = 2.6 \times 10^{-3}$ eV^2 [19]. At the moment no information at all about the CP phases or relative CP parities exists.

Regarding the total neutrino mass scale, only upper limits exist. Three different observables are at one's disposal, the effective Majorana mass $\langle m \rangle$ as measurable in neutrinoless double beta decay, the sum of neutrino masses Σ as testable through cosmology and the mass parameter m_{ν_e} as testable in direct kinematical experiments. Their definitions and current limits read

$$\begin{aligned}
&\langle m \rangle \equiv \Big|\sum U_{ei}^2 m_i\Big| \lesssim 1 \text{ eV [21]},\\
&\Sigma \equiv \sum |m_i| < 1.01 \text{ eV [22]},\\
&m_{\nu_e} \equiv \sqrt{\sum \big|U_{ei}^2 m_i^2\big|} < 2.2 \text{ eV [23]}.
\end{aligned} \tag{5}$$

Regarding $\langle m \rangle$, a factor of ~ 3 for the uncertainty in the nuclear matrix element calculations was included. The Heidelberg–Moscow Collaboration gives—using the results of one specific calculation for the nuclear matrix elements—a limit of $\langle m \rangle < 0.35$ eV at 90% C.L. [21].

2.2. Theory

This Letter is supposed to analyze the impact of a traceless m_ν. There exists a very simple and phenomenologically highly interesting explanation for this possibility [6]. The neutrino mass matrix is given by the see-saw mechanism [24] in general as

$$m_\nu = M_L - m_D M_R^{-1} m_D^T, \tag{6}$$

where m_D is a Dirac mass matrix and M_R (M_L) a right-handed (left-handed) Majorana mass matrix. In $SO(10)$ models, choosing Higgs fields in **10** and $\overline{\mathbf{126}}$ and the B–L breaking being performed by a **126** Higgs, one can write (see, e.g., [25]):

$$\begin{aligned}
&M_L = Y_{126} v_L, && M_R = Y_{126} v_R,\\
&m_{\rm down} = Y_{10} v_{10}^{\rm down} + Y_{126} v_{126}^{\rm down}, && m_{\rm lep} = Y_{10} v_{10}^{\rm down} - 3 Y_{126} v_{126}^{\rm down},\\
&m_{\rm up} = Y_{10} v_{10}^{\rm up} + Y_{126} v_{126}^{\rm up}, && m_D = Y_{10} v_{10}^{\rm up} - 3 Y_{126} v_{126}^{\rm up},
\end{aligned} \tag{7}$$

where $m_{\rm down\ (lep)}$ are the down quark (charged lepton), $m_{\rm up\ (D)}$ the up quark (Dirac) mass matrices, Y_{10} and Y_{126} are the Yukawa coupling matrices and $v_{10,126}^{\rm down\ (up)}$ are the vevs of the corresponding Higgs fields. The vevs corresponding to the Majorana mass matrices are denoted v_L and v_R. From (7) one finds $4Y_{126} = (m_{\rm down} - m_{\rm lep})/v_{10}^{\rm down}$. Suppose now that the first term in the see-saw formula (6) dominates. The mass matrix reads in this case:

$$m_\nu = Y_{126} v_L = (m_{\rm down} - m_{\rm lep}) \frac{v_L}{4 v_{10}^{\rm down}}. \tag{8}$$

Suppose now that $m_{\rm down}$ and $m_{\rm lep}$ are hierarchical, i.e., they contain small off-diagonal entries and the diagonal entries correspond roughly to the down quark and charged lepton masses, respectively. Then, the 23 sector of m_ν is diagonalized by

$$\tan 2\theta_{23} \propto \frac{1}{m_b - m_\tau}. \tag{9}$$

This mixing becomes maximal when b–τ unification takes place, i.e., $m_b = m_\tau$. This simple and appealing argument was first given in [10]. In [11] the idea was generalized to the full 3 flavor case and shown to be fully consistent with existing neutrino data.

Here we wish to emphasize that due to the same fact, b–τ unification, the trace of m_ν is proportional to $m_b - m_\tau$ and therefore, to a good precision, the trace vanishes [6]. We can quantify the smallness of the trace as

$$\mathrm{Tr}\, m_\nu \simeq \frac{(m_s - m_\mu) v_L}{4 v_{10}^{\rm down}} \simeq 0.025 \left(\frac{v_L}{\rm eV}\right)\left(\frac{\rm GeV}{v_{10}^{\rm down}}\right) \mathrm{eV}. \tag{10}$$

Here, $m_s \simeq 0.2$ GeV and $m_\mu \simeq 0.1$ GeV are the masses of the strange quark and muon, respectively. For the typical values of [26] $v_L \lesssim 0.1$ eV and $v_{10}^{\rm down} \gtrsim 1$ GeV we can expect the trace to be less than 10^{-3} eV. We shall take the fact as the starting point of our purely phenomenological analysis. Note that most of our results should be a specific case of a more detailed, but model-dependent analysis as performed in [11].

3. The CP conserving case

We shall investigate now the consequences of the requirement $\mathrm{Tr}\, m_\nu = 0$ on the mass states in the CP conserving case.

3.1. Normal hierarchy

Allowing for arbitrary relative signs of the mass states m_i with the convention $|m_3| > |m_2| > |m_1|$, the condition $m_1 + m_2 + m_3 = 0$ together with the experimental constraints of $\Delta m^2_{32} = \Delta m^2_{\rm A} = 2.6 \times 10^{-3}$ eV^2 and $\Delta m^2_{21} = \Delta m^2_\odot = 7.1 \times 10^{-5}$ eV^2 is solved by

$$m_1 = 0.0290 \text{ eV } \simeq m_2 = 0.0302 \text{ eV} \quad \text{and} \quad m_3 = -0.0593 \text{ eV } \simeq -2 m_2. \tag{11}$$

The numbers of course coincide with the ones presented in [7]. The characteristic relation $|m_3| \simeq 2|m_2| \simeq 2|m_1|$ holds as long as $\mathrm{Tr}\, m_\nu \lesssim 10^{-3}$ eV. The mass spectrum corresponds to a "partially degenerate" scheme.

The different relative signs of the mass states correspond to the $(- - +)$ configuration, for which $\alpha = 2\beta = \pi$. The effective mass $\langle m \rangle$ reads for these values and for $|m_3| \simeq 2|m_2| \simeq 2|m_1|$

$$\langle m \rangle \simeq \frac{|m_3|}{4} (3 \cos 2\theta_{13} - 1), \tag{12}$$

which is independent on $\tan^2\theta_{12}$. Varying θ_{13} leads to values of 0.025 eV $\lesssim \langle m \rangle \lesssim 0.030$ eV, thus predicting a very narrow range within the reach of running and future experiments [27].

Direct kinematical measurements will have to measure

$$m_{\nu_e} \simeq \frac{|m_3|}{2} \sqrt{1 + 4 \sin^2\theta_{13}} \simeq (0.030, \ldots, 0.032) \text{ eV}, \tag{13}$$

which is one order of magnitude below the limit of the future KATRIN experiment [28].

The sum of the absolute values of the neutrino masses is $\Sigma \simeq 0.12$ eV. As shown in [29], this is the lowest value (at 95% C.L.) measurable by combining data from the PLANCK satellite experiment and the sloan digital sky survey. Galaxy surveys one order of magnitude larger could reduce this limit by a factor of two [29] and thus test the prediction.

We turn now to a simple form of the mass matrix that accomplishes the requirement of being traceless. We concentrate on mass matrices with three parameters, sizable U_{e3} and no zero entries. For hierarchical neutrinos one might expect a quasi-degenerated and dominant 23 block of the mass matrix. Thus, one is lead to propose

$$m_\nu = \begin{pmatrix} -a & \epsilon_1 & \epsilon_2 \\ \cdot & a/2 & 3a/2 \\ \cdot & \cdot & a/2 \end{pmatrix}, \tag{14}$$

where $\epsilon_{1,2} \ll a$. Note that with $\epsilon_i = 0$ the mixing angles are $\theta_{23} = \pi/4$, $\theta_{13} = 0$ and $\tan\theta_{12} = 1/\sqrt{2}$, which is a widely discussed scheme [30]. We find with the mass matrix (14) that the mass states are

$$m_3 \simeq 2a, \qquad m_2 \simeq -a - \frac{\epsilon_1 - \epsilon_2}{\sqrt{2}}, \qquad m_2 \simeq -a + \frac{\epsilon_1 - \epsilon_2}{\sqrt{2}}, \tag{15}$$

and the observables are given by

$$\begin{aligned} &\Delta m_{\mathrm{A}}^2 \simeq 3a^2, \qquad \Delta m_{\odot}^2 \simeq 2\sqrt{2}\,a(\epsilon_1 - \epsilon_2), \\ &\tan 2\theta_{12} \simeq 6\sqrt{2}\,a \frac{\epsilon_1 - \epsilon_2}{(\epsilon_1 + \epsilon_2)^2}, \qquad \sin\theta_{13} \simeq \frac{1}{3\sqrt{2}\,a}(\epsilon_1 + \epsilon_2), \end{aligned} \tag{16}$$

together with maximal atmospheric mixing. For $\epsilon_1 = \epsilon_2 \neq 0$ the solar mixing angle vanishes. Comparing the last two equations with the data from Eq. (4), one finds that $a^2 \simeq 10^{-3}$ eV^2, $\epsilon_1 - \epsilon_2 \simeq 10^{-3}$ eV and $\epsilon_1 + \epsilon_2 \simeq 10^{-2}$ eV in order to reproduce the observations. It is seen that $|U_{e3}|$ should be sizable; we can express this element in terms of the other observables as

$$U_{e3}^2 \simeq \frac{1}{4} \frac{\Delta m_{\odot}^2}{\Delta m_{\mathrm{A}}^2} \frac{1 - \tan^2\theta_{12}}{\tan\theta_{12}}, \tag{17}$$

which becomes smaller, the larger the solar neutrino mixing angle θ_{12} becomes. Inserting the data from Eq. (4) in the right-hand side of the equation, the range of U_{e3}^2 is found for the LMA-I (LMA-II) case to lie between 0.0007 (0.002) and 0.0022 (0.04) in accordance with its current limit. The best-fit point predicts $U_{e3}^2 \simeq 0.0056$. The effective mass is given by

$$\langle m \rangle = a \simeq \sqrt{\Delta m_{\mathrm{A}}^2/3} \sim 0.03 \text{ eV}, \tag{18}$$

where we inserted the best-fit value of Δm_{A}^2. The allowed range of $\langle m \rangle$ lies between 0.022 eV and 0.036 eV, with a best-fit prediction of 0.029 eV. Both observables should thus be measurable with the next round of experiments. An alternative formulation of the correlation of observables reads

$$U_{e3}^2 \simeq \frac{\Delta m_{\odot}^2}{12\langle m \rangle^2} \frac{1 - \tan^2\theta_{12}}{\tan\theta_{12}}, \tag{19}$$

which could be used as a further check if both $\langle m \rangle$ and U_{e3}^2 were measured.

The question arises if the results are stable against radiative corrections. As known, the 12 sector is unstable for quasi-degenerate neutrinos with equal relative CP parity [31], which is what happens here. The effect of radiative corrections can be estimated by multiplying the $\alpha\beta$ element of m_ν with a term $(1 + \delta_\alpha)(1 + \delta_\beta)$, where

$$\delta_\alpha = c \frac{m_\alpha^2}{16\pi^2 v^2} \ln \frac{M_X}{m_Z}. \tag{20}$$

Here m_α is the mass of the corresponding charged lepton, $M_X \simeq 10^{16}$ GeV and $c = -(1 + \tan^2\beta)$ (3/2) in case of the MSSM (SM). We checked numerically that for the SM there is no significant change of θ_{12} and $\Delta m_{\odot}^2$ but for the MSSM and $\tan\beta \gtrsim 20$ the results become unstable. Also, the relation between $|U_{e3}|$ and the other observables remains its validity for the SM and for the MSSM with $\tan\beta \lesssim 20$.

One can relax the traceless condition a bit by adding a term proportional to $\mathbb{1} \times \xi/3$ to m_ν, where $\xi = \mathrm{Tr}\, m_\nu$. The mixing angles are of course unaffected by this term but the masses are changed by $m_i \to m_i + \xi/3$. Thus, the new mass squared differences read

$$\Delta m_{\odot}^2 \simeq 2\sqrt{2}\,(\epsilon_1 - \epsilon_2)(a - \xi/3) \quad \text{and} \quad \Delta m_{\mathrm{A}}^2 \simeq 3a\left(a + \frac{2}{3}\xi\right). \tag{21}$$

The correlation of the observables U_{e3} and $\langle m\rangle$ also changes, it is now given by

$$U_{e3}^2 \simeq \frac{1}{4}\Delta m_\odot^2 \frac{1-\tan^2\theta_{12}}{\tan\theta_{12}} \frac{1}{\Delta m_A^2 + \xi(\xi - \sqrt{\xi^2 + 3\Delta m_A^2})},$$
$$\langle m\rangle = a - \xi/3 \simeq \frac{1}{3}\left(\sqrt{\xi^2 + 3\Delta m_A^2} - 2\xi\right). \tag{22}$$

For $\xi = 0$ the previous two equations reproduce (16)–(18). The formula for the correlations of the observables, Eq. (19), holds also in the case of $\xi \neq 0$. As long as ξ does not exceed 10^{-3} eV, the corrections due to $\mathrm{Tr}\, m_\nu = \xi \neq 0$ increase (reduce) the predictions for U_{e3}^2 ($\langle m\rangle$) by $\sim \xi/\sqrt{\Delta m_A^2} \simeq 3\%$.

3.2. Inverted hierarchy

Allowing for arbitrary relative signs of the mass states m_i together with the convention $|m_2| > |m_1| > |m_3|$, the condition $m_1 + m_2 + m_3 = 0$ with the experimental constraints of $\Delta m_{13}^2 = \Delta m_A^2 = 2.6 \times 10^{-3}$ eV^2 and $\Delta m_{21}^2 = \Delta m_\odot^2 = 7.1 \times 10^{-5}$ eV$_2$ is solved by

$$m_2 = 0.0517 \text{ eV} \simeq -m_1 = 0.0510 \text{ eV} \quad \text{and} \quad m_3 = -0.0007 \text{ eV}. \tag{23}$$

The characteristic relation $|m_2| \simeq |m_1| \gg |m_3|$ holds as long as $\mathrm{Tr}\, m_\nu \lesssim 10^{-2}$ eV.

The signs of the mass states correspond to the $(+--)$ configuration for which $\alpha = \beta = \pi/2$. The effective mass then reads for $|m_2| \simeq |m_1| \gg |m_3|$

$$\langle m\rangle \simeq |m_2| \cos 2\theta_{12} \cos^2\theta_{13} \simeq |m_2| \frac{1-\tan^2\theta_{12}}{1+\tan^2\theta_{12}}, \tag{24}$$

which lies between 0.005 and 0.028 eV, thus predicting a range with the upper (lower) limit within (outside) the reach of running and future experiments [27]. The lower limit is however reachable by the 10t version of the GENIUS [32] project. The best-fit prediction is $\langle m\rangle \simeq 0.020$ eV. In contrast to the normal mass ordering, $\langle m\rangle$ has a crucial dependence on $\tan^2\theta_{12}$ and thus a rather large allowed range.

The mass measured in direct kinematical experiments is $m_{\nu_e} \simeq |m_2| \simeq 0.05$ eV, which is larger than the corresponding quantity in the normal hierarchy but still almost one order of magnitude below the limit of the future KATRIN experiment.

The sum of the absolute values of the neutrino masses is $\Sigma \simeq 0.10$ eV, lower than the corresponding quantity in the normal hierarchy and thus still requiring larger galaxy surveys, as shown in [29].

We present again a simple 3 parameter mass matrix with the traceless feature, no zero entries and non-vanishing U_{e3}. One is naturally lead to propose

$$m_\nu = \begin{pmatrix} -a & b & -b \\ \cdot & a/2 - \eta & -a/2 \\ \cdot & \cdot & a/2 + \eta \end{pmatrix}, \tag{25}$$

where $b > a > \eta$. We find with the mass matrix (14) that the mass states for $b^2 \gg \eta^2$ are

$$m_{2,1} \simeq \pm \frac{8b^4 + a^2(4b^2 + (1 \pm \sqrt{1 + 2b^2/a^2})\eta^2)}{4b^2\sqrt{a^2 + 2b^2}} \quad \text{and} \quad m_3 \simeq \frac{-a}{2b^2}\eta^2. \tag{26}$$

The observables are found to be

$$\tan 2\theta_{12} \simeq \sqrt{2}\,\frac{b}{a}, \qquad \sin\theta_{13} \simeq \frac{\eta}{\sqrt{2}\,b},$$
$$\Delta m_A^2 \simeq a^2 + 2b^2, \qquad \Delta m_\odot^2 \simeq \frac{a}{b^2}\sqrt{a^2 + 2b^2}\,\eta^2. \tag{27}$$

Again, the observed values of the quantities are easy to reproduce with, in this case, e.g., $b > a > \eta \sim 0.01$ eV. There is again a simple correlation of the observables, namely

$$U_{e3}^2 \simeq \frac{1}{2} \frac{\Delta m_\odot^2}{\Delta m_A^2} \frac{1+\tan^2\theta_{12}}{1-\tan^2\theta_{12}}. \tag{28}$$

Putting again the data from Eq. (4) in the right-hand side of this equation leads to $U_{e3}^2 \gtrsim 0.013$ with a best-fit prediction of $U_{e3}^2 \simeq 0.036$. For large values of $\Delta m_\odot^2$, i.e., in the less favored LMA-II solution, which corresponds to $\Delta m_\odot^2 \gtrsim 10^{-4}$ eV^2, the value of U_{e3} is above its current experimental limit. Comparing the expressions for U_{e3} in the normal (Eq. (17)) and inverted (Eq. (28)) ordering leads to the observation that for the first case the value is smaller by a factor of $\simeq 1/2(1-\tan^2\theta_{12})^2/((1+\tan^2\theta_{12})\tan\theta_{12}) \lesssim 0.35$.

The effective mass is given by

$$\langle m \rangle = a \simeq \frac{\Delta m_\odot^2}{2\sqrt{\Delta m_A^2}\, U_{e3}^2} = \sqrt{\Delta m_A^2}\, \frac{1-\tan^2\theta_{12}}{1+\tan^2\theta_{12}} \sim 0.02 \text{ eV}. \tag{29}$$

Comparing this result with our prediction for $\langle m \rangle$ in the normal mass ordering, Eq. (16), one finds that the inverted mass hierarchy predicts an effective mass smaller than a factor $\sqrt{3}\tan 2\theta_{12} \gtrsim 4$. This is larger than the typical uncertainty of the nuclear matrix elements that usually tends to spoil extraction of information from neutrinoless double beta decay. If both, $\langle m \rangle$ and U_{e3} are measured, one can further check the mass matrix by the relation

$$U_{e3}^2 \simeq \frac{\Delta m_\odot^2}{2\langle m \rangle \sqrt{\Delta m_A^2}}. \tag{30}$$

We checked numerically that the results are stable under radiative corrections in the SM and in the MSSM for $\tan\beta \lesssim 50$.

One can again relax the traceless condition through a contribution $\mathbb{1} \times \xi/3$ to m_ν, where $\xi = \operatorname{Tr} m_\nu$. The new mass squared differences are

$$\Delta m_\odot^2 \simeq \frac{a\eta^2 + \frac{4}{3}b^2\xi}{b^2}\sqrt{a^2+2b^2} \quad \text{and} \quad \Delta m_A^2 \simeq a^2 + 2b^2 - \frac{2}{3}\sqrt{a^2+2b^2}\,\xi. \tag{31}$$

Again, the correlation of the observables U_{e3} and $\langle m \rangle$ changes, one finds

$$\begin{aligned} U_{e3}^2 &\simeq \frac{9}{2}\frac{1+\tan^2\theta_{12}}{1-\tan^2\theta_{12}} \frac{\Delta m_\odot^2 - \frac{4}{9}\xi(\xi + \sqrt{9\Delta m_A^2 + \xi^2})}{(\xi + \sqrt{9\Delta m_A^2 + \xi^2})^2}, \\ \langle m \rangle &= a - \xi/3 \simeq \frac{1}{3(1+\tan^2\theta_{12})}\Big((1-\tan^2\theta_{12})\sqrt{\xi^2 + 9\Delta m_A^2} - 2\tan^2\theta_{12}\Big). \end{aligned} \tag{32}$$

For $\xi = 0$ the results for exact zero trace given above are re-obtained.

Interestingly, the same mass matrix, Eq. (25), has been found in [5]. In this Letter a local horizontal $SU(2)$ symmetry has been applied to the charged leptons. A consequence was a vanishing determinant of m_ν and an inverted hierarchy for the neutrino masses (i.e., $m_3 = 0$) with opposite signs for m_2 and m_1. In this case, both the trace and the determinant of m_ν are vanishing, which explains that our results are identical to the ones in [5].

To put this section in a nutshell, the requirement of a vanishing trace of m_ν leads in the CP conserving case to values of $\langle m \rangle$ larger in the NH than in the IH. Due to the dependence on $\tan\theta_{12}$, $\langle m \rangle$ in case of IH has a broad range. Simple mass matrices were proposed which reproduce the values found by the traceless condition and in addition predict larger U_{e3} in the IH case. Relaxing the traceless condition does not significantly change the predicted values as long as the trace stays below the expected 10^{-3} eV.

4. The CP violating case

Now we shall investigate the more realistic case of CP violation and the consequences of the traceless m_ν condition. Within the parametrization (2) one finds—using Eq. (1)—for the trace of m_ν that

$$\mathrm{Tr}\, m_\nu \simeq m_1 + m_2 e^{2i\alpha} + m_3 e^{2i(\beta+\delta)}. \tag{33}$$

Terms of order $\sin^2\theta_{13}$ were neglected, which can be shown to be a justified approximation. The condition of zero trace holds for the real and imaginary part of $\mathrm{Tr}\, m_\nu$, i.e.,

$$\begin{aligned} &m_1 + m_2 \cos 2\alpha + m_3 \cos 2(\beta+\delta) = 0, \\ &m_2 \sin 2\alpha + m_3 \sin 2(\beta+\delta) = 0. \end{aligned} \tag{34}$$

The minimal values of m_1 or m_3 that fulfill the condition (34) are the ones that correspond to the CP conserving case discussed in the previous section. As a check, one can convince oneself that for $\delta = 0$ and $m_1 = m_2 = m_3/2$ the solution of the two equations in (34) is given by $\alpha = 2\beta = \pi$ while for $\delta = 0$ and $m_1 = m_2 \gg m_3 \simeq 0$ one finds that $\alpha = \pi/2$, which is in accordance with the previous section. This means that in case of the normal hierarchy and the LMA-I (LMA-II) solution a lower limit on the neutrino mass of 0.019 (0.021) eV can be set, which is obtained by inserting the lowest allowed Δm^2_{A} and the largest $\Delta m^2_\odot$. In case of inverted hierarchy, one finds that $|m_3| \geqslant 0.0013$ (0.0024) eV for the LMA-I (LMA-II) solution.

Due to the zero trace condition one can write

$$m_1^2 = m_2^2 + m_3^2 + 2 m_2 m_3 \cos\phi, \quad \text{where} \quad \phi = 2(\alpha - \beta - \delta). \tag{35}$$

Interestingly, this implies that in the expressions for Σ and m_{ν_e} the phases appear. In particular,

$$m^2_{\nu_e} = \frac{1}{1+\tan^2\theta_{12}} \left(m_3^2 + m_2^2 \left(1+\tan^2\theta_{12}\right) + 2 m_2 m_3 \cos\phi \right) + m_3^2 \sin^2\theta_{13}. \tag{36}$$

For quasi-degenerate neutrinos, i.e., $m_3 \simeq m_2 \simeq m_1 \equiv m_0$, one finds from Eq. (35) that $\cos\phi = -1/2$ or equivalently $\alpha - \beta - \delta \simeq \pm\pi/3 \pm n\pi$. Thus, quasi-degenerate neutrinos and the zero trace condition demand non-trivial correlations between the CP phases.

Applying the condition $\mathrm{Tr}\, m_\nu = 0$ to Eq. (35) and inserting it in the expression for $\langle m \rangle$ one finds

$$\langle m \rangle \simeq \frac{1}{1+\tan^2\theta_{12}} \sqrt{m_3^2 + m_2\left(1-\tan^2\theta_{12}\right)\left(m_2\left(1-\tan^2\theta_{12}\right) + 2 m_3 \cos\phi\right)}, \tag{37}$$

where we neglected $\sin^2\theta_{13}$. Courtesy of the zero trace condition, $\langle m \rangle$ depends effectively only on one phase. The CP conserving cases in the previous section should come as special cases of the last formula. Indeed, for $\delta = 0$, $m_2 = m_3/2$ and $\alpha = 2\beta = \pi$ one recovers Eq. (12) and for $\delta = 0$ and $m_3 = 0$ one re-obtains Eq. (24). For quasi-degenerate neutrinos $m_0 \equiv m_2 \simeq m_3$ the above formula simplifies. Then, since $\cos\phi \simeq -1/2$:

$$\langle m \rangle \simeq m_0 \frac{\sqrt{1+\tan^2\theta_{12}(\tan^2\theta_{12}-1)}}{1+\tan^2\theta_{12}}, \tag{38}$$

which can be used to set an upper limit on m_0. For $\langle m \rangle \lesssim 1$ eV we have $m_0 \lesssim 1.96$ eV with a best-fit limit of 1.67 eV. Therefore, the zero trace condition implies a limit stronger than the one stemming from direct kinematical experiments. Using the less conservative limit given by the Heidelberg–Moscow Collaboration, the above limits are reduced by a factor of roughly 2.9 and the limits come nearer to the ones from cosmological observations. To be precise, for $\langle m \rangle < 0.35$ eV one finds $m_0 \lesssim 0.69$ eV and, for the best-fit value, $m_0 \lesssim 0.58$ eV. The values are testable by the KATRIN experiment. Thus, together with the lower limit (about 0.02 eV for NH and 0.002 eV for IH) from the beginning of this section, a neutrino mass window is defined.

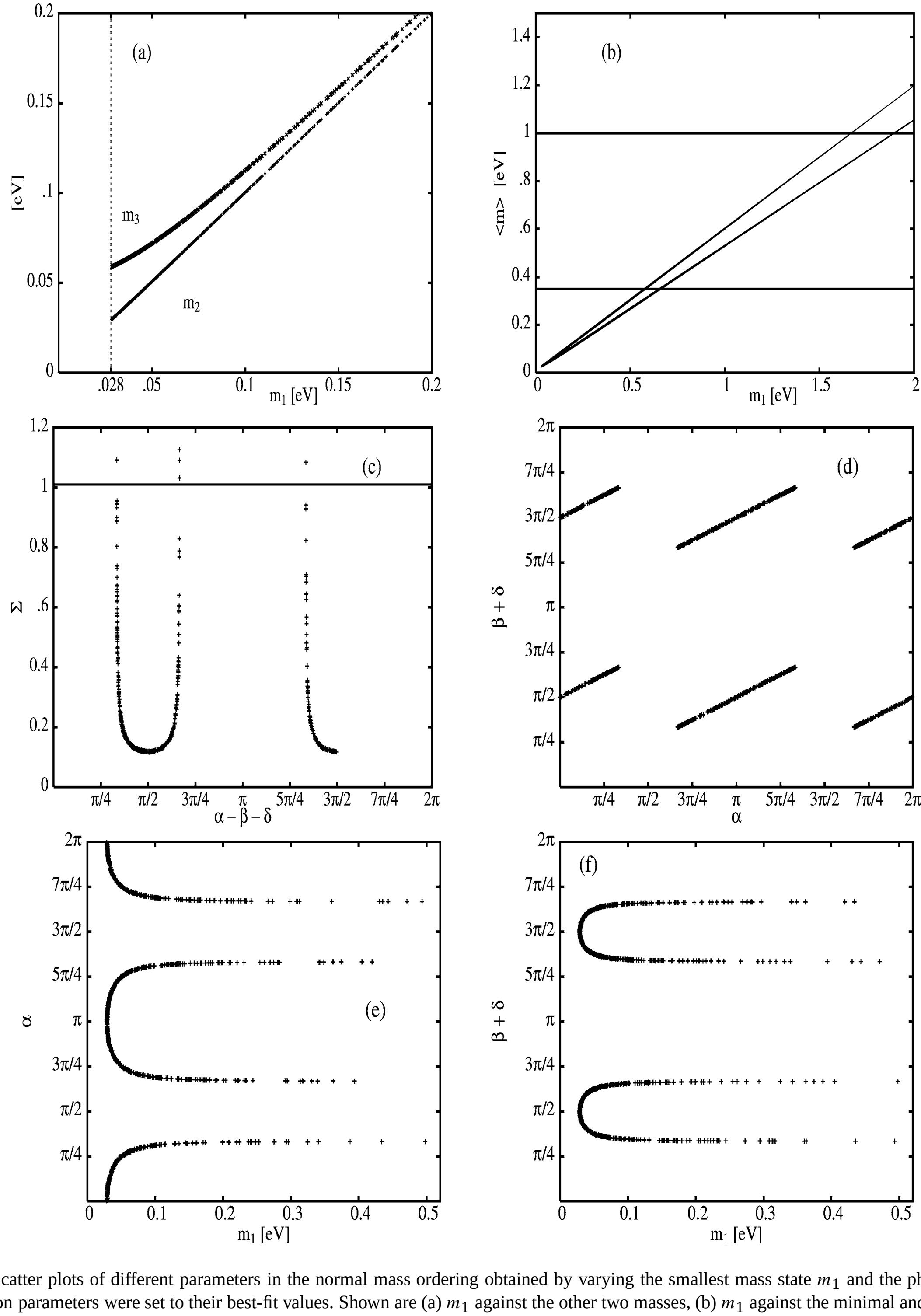

Fig. 1. Scatter plots of different parameters in the normal mass ordering obtained by varying the smallest mass state m_1 and the phases. The oscillation parameters were set to their best-fit values. Shown are (a) m_1 against the other two masses, (b) m_1 against the minimal and maximal value of $\langle m \rangle$ (given by varying θ_{13}), (c) $\alpha - \beta - \delta$ against Σ, (d) α against $\beta + \delta$, (e) m_1 against α and (f) m_1 against $\beta + \delta$.

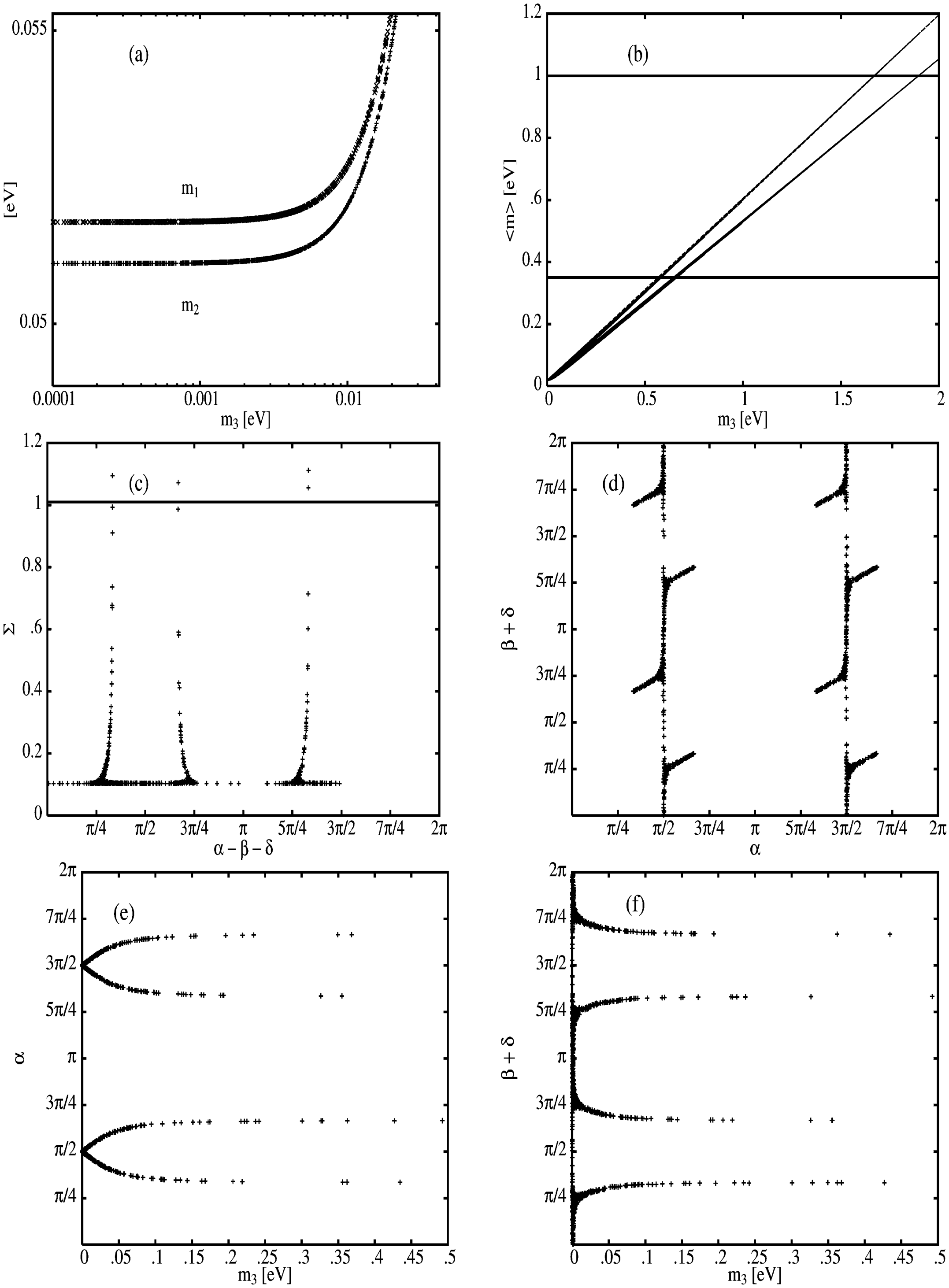

Fig. 2. Same as previous figure for the inverted mass ordering.

One can compare the predictions for $\langle m \rangle$ in case of zero trace with the ones in case of zero determinant [3]. This corresponds for NH (IH) to zero m_1 (m_3), which results in particularly simple forms of $\langle m \rangle$, see [3] for details. Regarding the inverted hierarchy, we already commented that in case of an opposite relative sign of the two quasi-degenerate neutrinos and a very small m_3 both the trace and the determinant vanish and the situation is identical. We use the data from Eq. (4) for our predictions. For the normal mass ordering strong cancellations are possible [18,33], and $\langle m \rangle$ is in general predicted to be below 0.01 eV. In case of the inverted mass ordering, $\langle m \rangle$ lies between 0.004 and 0.034 eV, independent on $\sin^2 \theta_{13}$. Unlike the zero trace case, the zero determinant conditions allows no statements about the phases, at least not before the limit on $\langle m \rangle$ is significantly improved.

We also performed a numerical analysis of the zero trace condition. For this exercise the mass squared differences and solar neutrino mixing angle were fixed to their best-fit points and the smallest neutrino mass as well as the phases were varied within their allowed range. The results in the form of scatter plots for the normal hierarchy are shown in Fig. 1 and for the inverted scheme in Fig. 2. One recognizes for example in Figs. 1(c), 2(c) the correlation of Σ with $\alpha - \beta - \delta$ as implied by Eq. (35). For the inverted hierarchy, the spread of the phases is rather different from the case of normal hierarchy. This can be understood from the fact that for small m_3 the dependence on $\beta + \delta$ practically vanishes.

5. Summary and conclusions

The condition of a zero trace of the neutrino mass matrix m_ν was reanalyzed in case of CP conservation and violation for both possible mass orderings. The motivation for this purely phenomenological analysis was given by exact b–τ unification in connection to the non-canonical type II see-saw mechanism in $SO(10)$ models. This situation has recently gathered renewed attention because of its ability to produce large atmospheric neutrino mixing in a simple way. In case of CP conservation, the values of the neutrino masses and their relative CP parities are fixed and allow to give simple expressions for the effective mass as measurable in neutrinoless double beta decay. The masses are given by $m_1 \simeq m_2 \simeq -m_3/2 \simeq 0.03$ eV for the normal mass ordering and $\sqrt{\Delta m_{\rm A}^2} \simeq m_2 \simeq -m_1 \gg -m_3$ for the inverted mass ordering. In case of the normal hierarchy, $\langle m \rangle$ does not depend on the solar neutrino mixing angle and is predicted to be around 0.03 eV. In case of inverted hierarchy, $\langle m \rangle$ depends rather strongly on the solar neutrino mixing angle and its range is between 0.005 and 0.03 eV; the best-fit prediction is 0.02 eV. The presence of CP violation and therefore non-trivial values of the Majorana phases allows for larger values of the masses. In case of quasi-degenerate neutrinos a peculiar relation between the phases exists: $\alpha - \beta - \delta = \pm\pi/3 \pm n\pi$. The minimal values of the masses correspond to the CP conserving case and are in case of the normal (inverted) hierarchy roughly 0.02 (0.002) eV. The upper limit comes from non-observation of neutrinoless double beta decay and is for $\langle m \rangle < 0.35$ eV roughly 0.7 eV. Correlations of various parameters are possible, some of which are shown in Figs. 1 and 2.

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